

Problem Set I: Due Wednesday, January 20, 2016

- 1.) Derive by heuristic methods the thermal diffusivity and shear viscosity for a dilute gas of particles undergoing hard sphere interactions.
- 2.) Consider a heavy particle of mass M , radius d_2 in a gas of light particles of mass m , radius d_1 ($m \ll M$, $d_2 > d_1$) at temperature T .
 - a.) Estimate the mobility of the heavy particle.
 - b.) When will the heavy particle be fully deflected from its trajectory? What is the deflection length?
 - c.) When will the energy of the heavy particle equal that of the light background particle?
- 3.) Use the result from Problem 2 to estimate the time scale for thermal equilibration of ions of mass M_i and temperature T_i with electrons of mass m_e and T_e . Take $T_e > T_i$. Compare this time scale with the electron-electron and ion-ion collision times.

N.B.: For Problems 1-3, you may find it useful to consult the Supplementary Notes on Scaling Methods and to read “Qualitative Methods in Physical Kinetics and Hydrodynamics”, by V.P. Krainov.

- 4.) Consider a plasma for which $\underline{J} = \sigma \underline{E}$ with σ a constant, and for which the displacement current is negligible.
 - a.) Derive an equation for the magnetic field \underline{B} . What type of equation is this? Comment on the coefficient.
 - b.) Assuming $\underline{B} = B(x)e^{-i\omega t}$, what is the depth of penetration of the field into the plasma? This is the collisional skin depth.
 - c.) Calculate the energy dissipated by the penetration process.

- d.) Calculate the corresponding penetration depth for a collisionless plasma, retaining displacement current effects, as discussed in class. Assume $\omega < \omega_{pe}$.
- 5.) Consider a beam of electrons moving at constant speed $V_0 \hat{x}$. The beam is shot into a background plasma. The beam has density n . Take $n_b \ll n_0$, where n_0 is the density of the background plasma.
- a.) What are the dielectric function and wave frequency for waves in the beam alone? Discuss the energy of these waves. Why can the energy be negative?
- b.) Now derive the dielectric for plasma waves in the beam + plasma system. Discuss the result.
- c.) Take $n_b = n_0$. Investigate the stability of the system by going to a frame moving at $-(V_0/2)\hat{x}$.
- 6.) Derive the dispersion relation for an ion-acoustic wave for the case of warm ions. Take ions adiabatic and electrons isothermal. Why is this reasonable?
- 7.) Read the classic paper by P. Debye, posted under Supplementary Material. Write a 1-2 page summary. Be prepared to present this to the class. Be sure to answer:
- Why was Debye driven to confront screening processes?
 - What was the impact of screening on the colloidal aggregation process he was studying?

N.B.: You may also find it useful to consult the posted article by Chandrasekhar.